Principles of Macroeconomics: Introduction to Economic Growth

Class 3

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Overview

- Announcements:
  - You should be able to do LC 7 and GH 7 (due September 12th at 11:59pm)
  - Presentations
- Topics:
  - Living Standards Over Time
  - Analytical Tools for Growth
- Readings:
  - Chapter 9.1 (Comparing Economies), chapter 9.2 (Sources of Long-Run Growth)

What's Happened Over the Last 100 Years

- In 1900, the US had:
  - Life expectancy of $<$ 50 years
  - 10% of infants dying before 1
  - $>$ 90% of households having no electricity/telephone/ or car
  - $<$ 10% of adults completing high school
- In 2025, the US has:
  - Life expectancy of $>$ 78 years
  - $<$ 1% of infants dying before 1
  - 100% of households have access to electricity (World Bank)
  - $>$ 90% of adults completing high school
  - AND: access to air-conditions, dishwashers, passenger planes, skyscrapers, iPads, AI

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Another View of Progress

- In 1836, Nathan Rothschild was the richest person in the world
  - Perspective: he was worth 0.62% of Britain's GDP at the time
- Rothschild died from a bacterial infection
  - $<$ $50 of penicillin today would have likely been enough to save him

How Will We Measure Progress?

- Remember real GDP? What is real GDP?

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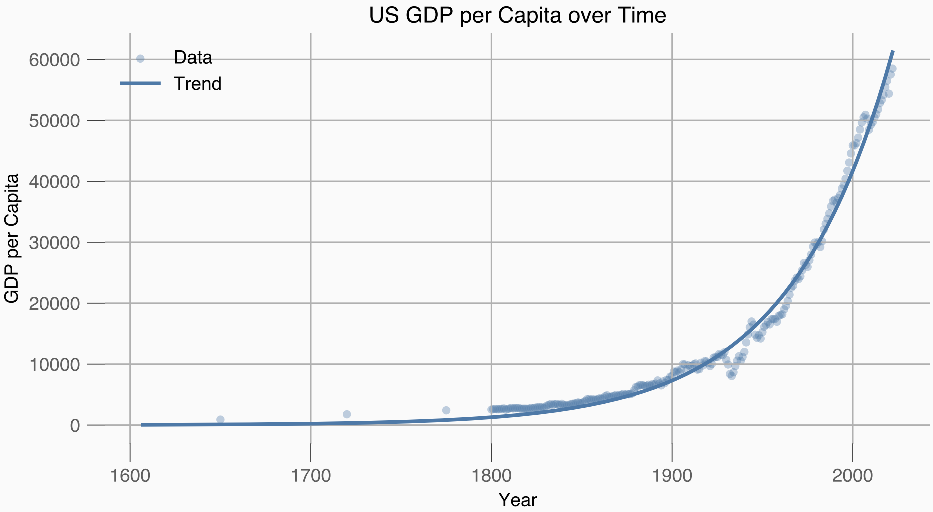
- We now want to account for real GDP per person:

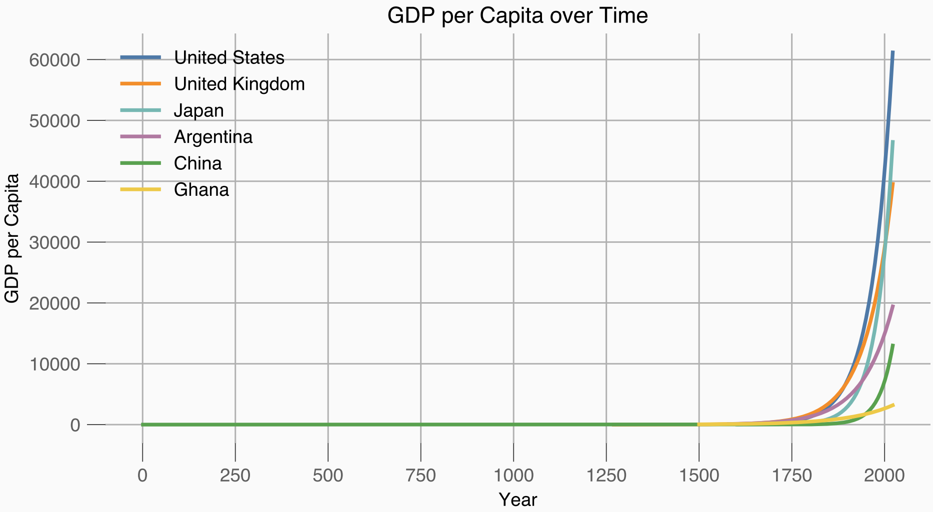
$$\text{RGDP}_{\text{person}} = \frac{\text{RGDP}}{\text{Population}}$$

- We will use this as our "standard of living"
  - $\uparrow \text{RGDP}_{\text{person}} \longrightarrow \uparrow$ consumption
  - $\uparrow \text{RGDP}_{\text{person}} \longrightarrow \uparrow$ Health
  - $\uparrow \text{RGDP}_{\text{person}} \longrightarrow \uparrow$ Life Expectancy
  - $\uparrow \text{RGDP}_{\text{person}} \longrightarrow \uparrow$ "Happiness"

The US: 1800 ≈ $2500 to 2022 ≈ $60,000



Maybe it's just the US?



Growth Rates

- Growth rate just means percentage change
- For example: let y be RGDP per person
- The growth rate of y from 2022 to 2023 would be:

$$\frac{y_{2023} - y_{2022}}{y_{2022}}$$

- Suppose that the growth rate of y is g. Then: $y_{t+1} = y_t(1 + g_t)$
- Essentially, we need two of the three variables in that equation:
  - If we have y_t and y_{t+1}, we can get g_t
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An Example

- World population was 6 billion in 2000
  - Set 2000 to be $t = 0$
  - Set 6 billion to be L_0
- Suppose that population growth is 2% every year for the next 100 years
- Question: What will the population be in 2100?

Constant Growth Rule

- We can use something called the Constant Growth Rule:

$$L_t = L_0(1 + g_L)^t \ \forall \ t = \{0, 1, \ldots, \infty\}$$

- So we know L_0 and we know n – we can easily solve for L_t:

$$L_t = 6 \text{ billion} \times (1 + 0.02)^{100} \approx 43.5 \text{ billion}$$

- Now what if we knew L_{100} and L_0?

- We could solve for n:

$$
\begin{aligned}
L_{100} &= L_0(1+g_L)^{100} \\
\frac{L_{100}}{L_0} &= (1+g_L)^{100} \\
\left(\frac{L_{100}}{L_0}\right)^{\frac{1}{100}} - 1 &= g_L
\end{aligned}
$$

Rule of 70

$$\#\text{ of years to double} = \frac{70}{\text{annual growth rate}}$$

- Suppose that population grows at 2% a year. Then population takes 35 years to double
  - We can check (do you remember your ln rules?):

$$\frac{L_t}{L_0} = (1.02)^t$$
$$\frac{2L_0}{L_0} = (1.02)^t$$
$$\ln(2) = t\ln(1.02)$$
$$35 \approx t$$

- Lesson: Small difference in growth rates $\longrightarrow$ big effects on time to double
  - 1% growth rate $\longrightarrow$ 70 years to double
  - 2% growth rate $\longrightarrow$ 35 years to double
  - 3% growth rate $\longrightarrow$ 23 years to double
- So a 1pp change in the growth rate could cause population to double once in your lifetime or 8x in your lifetime!

Rule of 70 Derivation

- Start from:

$$2 = (1+g)^t$$

- Use natural logs:

$$\ln(2) = t\ln(1+g)$$

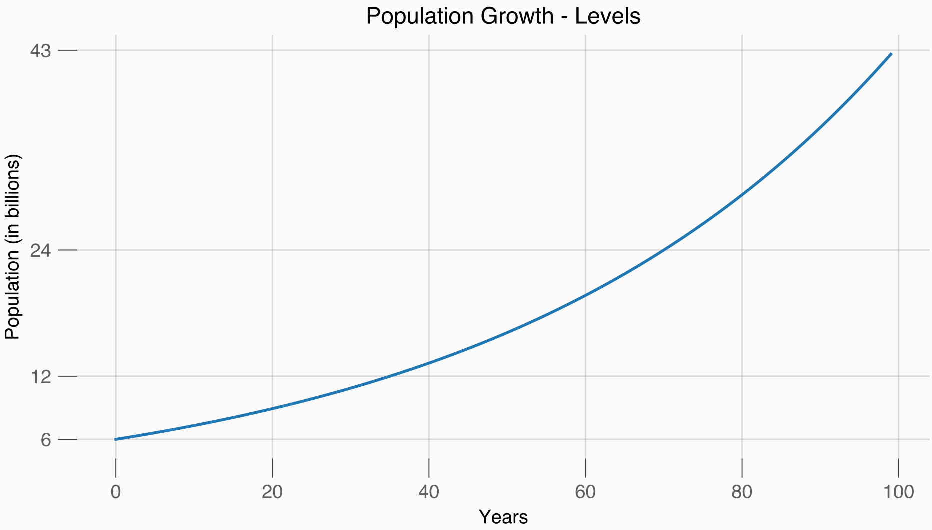
- We can approximate $\ln(1+g)$ as tg for small g

$$\begin{aligned} tg &= \ln(2) \\ t &\approx \frac{0.7}{g} = \frac{70}{100g} \end{aligned}$$

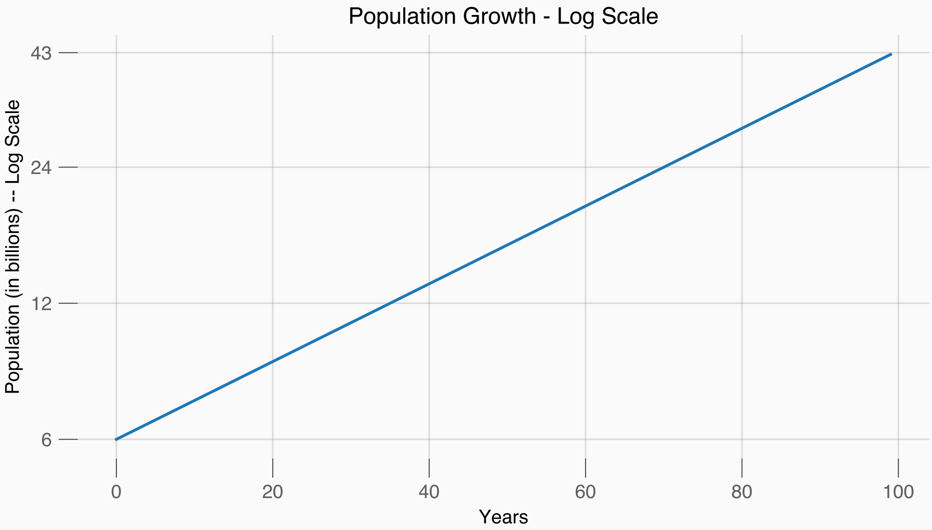
Ratio Scale

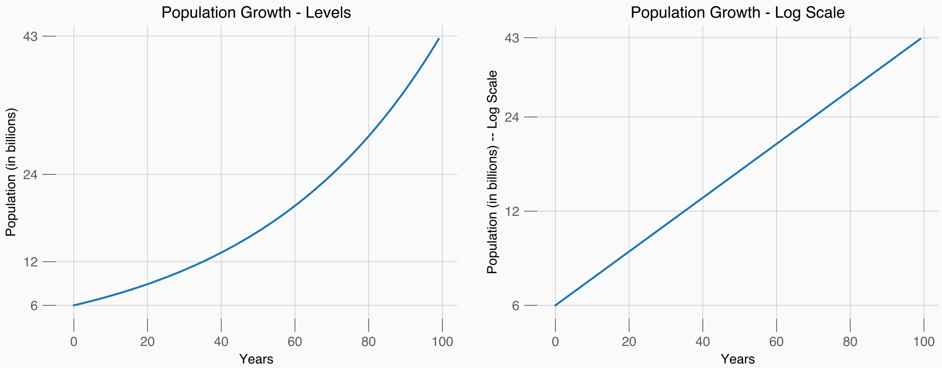
- Notice that we only need growth rates for time to double – levels are not needed!
- We can then plot two ways:
  1. Plot the level
  2. Plot the natural logs – this makes the ratio between doubling times equidistant (e.g. if we have "1, 2, 4, 8", log scale makes the distance between 1 and 2, 2 and 4, and 4 and 8 equal)

Option 1


Population Growth - Levels
Population (in billions)
43
24
12
6
0
20
40
60
80
100
Years

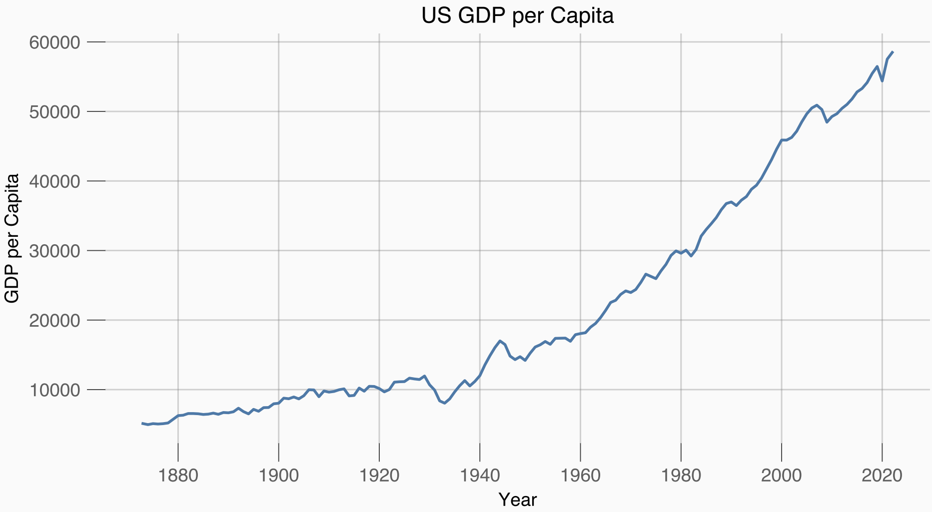
Option 2


Population Growth - Log Scale
Population (in billions) -- Log Scale
43
24
12
6
0
20
40
60
80
100
Years

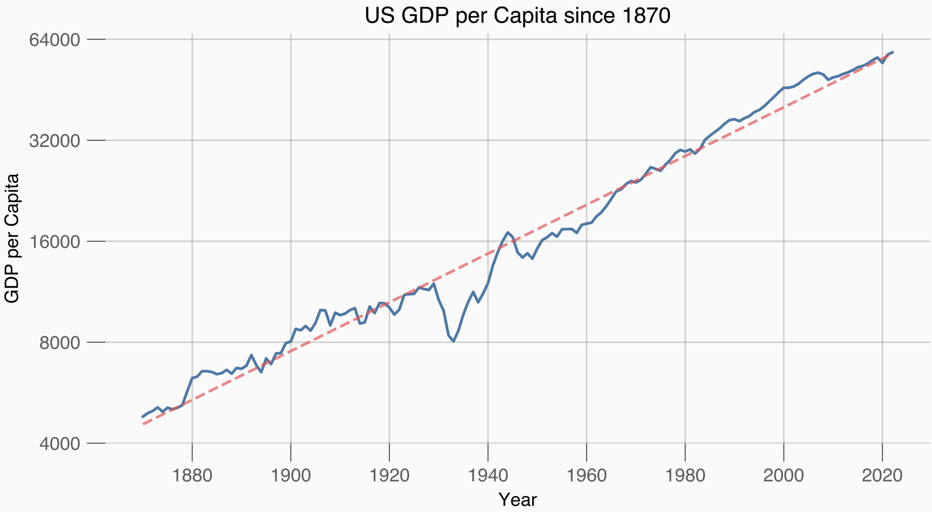


- The log-scale's slope is the growth rate
- Either way, we get the same total population at the end!

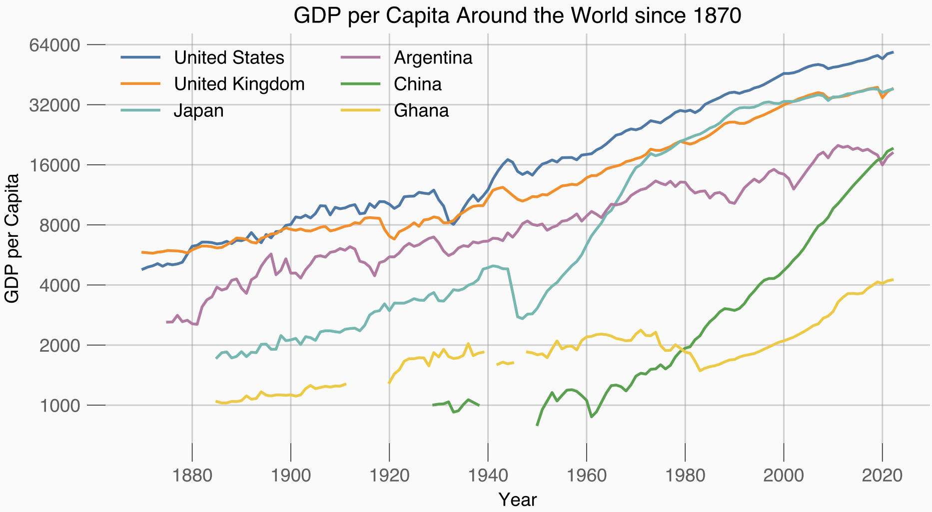
Returning to US RGDP per Capita



Almost 2% (1.8%) on Average!



Are Other Countries this Stable?



(1) Suppose that country A grows at 1.8% a year and, in year 0, has a GDP per capita of \$60,000. Suppose that country B grows at 4% a year and, in year 0, has a GDP per capita of \$15,000. In how many years will country B catch up to country A?

(2) Suppose country C grows at 1.2% a year and starts at a GDP per person of \$55,000. Suppose country D starts at a GDP per person of \$20,000. At which constant growth rate will country D overtake country C in 40 years?

(1) Remember the formula for constant growth: $y_A(t) = y_A(0)(1+g_A)^t$. We want to know when $y_A = y_B$. So set the growth formulas equal to each-other:

$$\begin{aligned}
y_A(0)(1+g_A)^t &= y_B(0)(1+g_B)^t \\
\ln(y_A(0)) + t\ln(1+g_A) &= \ln(y_B(0)) + t\ln(1+g_B) \\
\ln(y_A(0)) - \ln(y_B(0)) &= t\left(\ln(g_B) - \ln(g_A)\right) \\
\frac{\ln(y_A(0)) - \ln(y_B(0))}{\ln(g_B) - \ln(g_A)} &= t \\
64.84 &\approx t
\end{aligned}$$

(2) We can use the same formula, but now we are solving for g_D:

$$
\begin{aligned}
y_C(0)(1+g_C)^t &= y_D(0)(1+g_D)^t \\
\frac{y_C(0)}{y_D(0)}(1+g_C)^t &= (1+g_D)^t \\
\left[\frac{y_C(0)}{y_D(0)}\right]^{\frac{1}{t}}(1+g_C)^t &= 1+g_D \\
\left[\frac{y_C(0)}{y_D(0)}\right]^{\frac{1}{t}}(1+g_C)-1 &= g_D \\
3.79\% &\approx g_D
\end{aligned}
$$

(3) Suppose an economy experiences a recession once a decade where the GDP growth rate is -1%. What constant interest rate must the economy grow at for the other nine years for the economy to arithmetically average 1.8% growth for the decade?

(4) Suppose a benevolent social planner can select between two growth plans for the economy. In growth plan A, the planner makes the economy grow at 1.8% per year, every year, for 50 years. In growth plan B, the planner makes the economy grow at the rate from problem 3 for nine years every decade, but allows GDP per capita to fall 1% in the tenth year. The planner repeats this for 50 years. Which plan is better?

(5) At which growth rate under plan B would we be indifferent between plan A and plan B?

Solutions

(3) Use the arithmetic average:

$$\frac{9 \times r + (-1)}{10} = 1.8$$
$$9 \times r = 18 + 1$$
$$r = \frac{19}{9} \approx 2.11\%$$

(4) Let GDP per capita in period 0 be unity, without loss of generality. Then growth after 50 years under plan A is:

$$y_A(50) = (1 + 0.018)^{50} \approx 2.44$$

Under plan B, growth each decade is:

$$y_B(10) = (1.0211)^9 \times (1 - 0.01)^1 \approx 1.195$$

Compound this five times:

$$y_B(50) = 1.195^5 \approx 2.43$$

So there is a penalty to volatility

(5) Set the growth formulas equal:

$$1.018^{50} = \left((1 + r)^9 \times 0.99\right)^5$$

$$1.018^{10} = (1 + r)^9 \times 0.99$$

$$\left(\frac{1.018^{10}}{0.99}\right)^{\frac{1}{9}} - 1 = r$$

$$2.12\% \approx r$$

Summary

- Most of human history, all humans have lived in relative poverty
- Lots of progress in the last century
- Exponential growth is awesome
- Are other countries "catching up"? Why are some countries growing and some not?
- Thursday: Sources of Long Run Growth
  - Read chapter 9.2